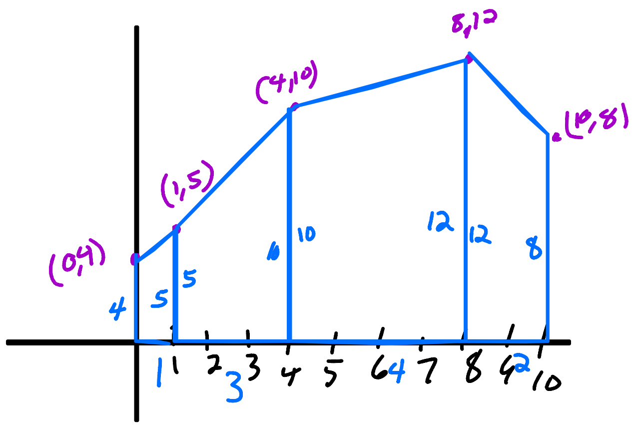
Period 3, Jan 24, 2024

4. A function f is continuous on the closed interval [0,10] and has values

x	0	1	4	8	10
$f(x)$	4	5	10	12	8

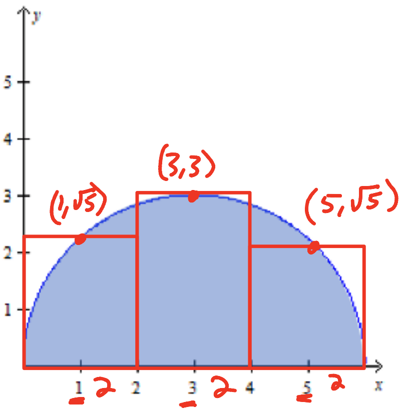
Find an approximation to $\int_0^{10} f(x)\,dx$ using a trapezoidal sum with the four subintervals $[0,1]$, $[1,4]$, $[4,8]$, and $[8,10]$.



$\frac{1}{2}(4+5)\cdot 1 + \frac{1}{2}(5+10)\cdot 3 + \frac{1}{2}(10+12)\cdot 4 + \frac{1}{2}(8+12)\cdot 2$

5. The graph of $g(x) = \sqrt{6x - x^2}$ is shown below.

x	y
1	$\sqrt{5} = \sqrt{6(1) - 1^2} = \sqrt{5}$
3	$3 = \sqrt{6(3) - 3^2} = \sqrt{9}$
5	$\sqrt{5} = \sqrt{6(5) - 5^2} = \sqrt{5}$



$[0,6]$

$\frac{6-0}{3} = \frac{6}{3} = 2$

a) Approximate the area under the graph of $g(x)$ from [0,6] using a Midpoint Riemann sum with 3 sub intervals of equal length $= 2$

$2\sqrt{5} + 3 \cdot 2 + 2\sqrt{5} = 4\sqrt{5} + 6$

b) Express the area under the graph of $g(x)$ as a definite integral

$$\int_0^6 \sqrt{6x - x^2}\, dx$$

c) Use technology to evaluate the integral.

d) Confirm the answer to (c) using geometry

semi-circle Radius $= 3$

$$\text{Area} = \frac{\pi r^2}{2} = \frac{3^2\pi}{2} = \frac{9\pi}{2} =$$

c

$$\int_0^6 \sqrt{6x - x^2}\, dx$$

$= 14.1371669412$

$y = 5x^3 + 6x^7 - 4$ when $x=1$ $y=7$

$$\frac{dy}{dx} = 15x^2 + 42x^6 + 0$$

$$\int (15x^2 + 42x^6)\,dx$$

$$15 \cdot \frac{1}{3} \cdot x^{2+1=3} + 42 \cdot \frac{1}{7} \cdot x^{6+1=7} + C$$

$y = 5x^3 + 6x^7 + C$ Point $(1,7)$

$$7 = 5(1)^3 + 6(1)^7 + C \Rightarrow y = 5x^3 + 6x^7 - 4$$

$$7 = 5 + 6 + C$$

$$-4 = C$$

$$\int \sec^2\theta \, d\theta = \tan\theta + C$$

$$\int \frac{e^x}{5+e^x}\,dx$$

$5 + e^x = a$

$da = 0 + e^x\,dx$

$$\frac{da}{e^x} = \frac{e^x\,dx}{e^x}$$

$$\frac{da}{e^x} = dx$$

$$\int \frac{\cancel{e^x}}{a} \cdot \frac{da}{\cancel{e^x}}$$

$$\int \frac{da}{a} = \ln|a| + C$$

$$\ln|5+e^x| + C$$

$$\int x\sqrt{2x-1}\,dx$$

$$2x-1=a \Rightarrow \frac{a+1}{2}=x$$

$$2\,dx = da$$

$$dx=\frac{da}{2}$$

$$\int x\sqrt{a}\,\frac{da}{2}$$

$$\int \frac{a+1}{2}\sqrt{a}\cdot\frac{da}{2}=\frac{1}{4}\int\left((a+1)\sqrt{a}\right)da$$

$$\frac{1}{4}\int\left(a^{\frac{3}{2}}+a^{\frac{1}{2}}\right)da$$

$$\frac{1}{4}\left[1\cdot\frac{2}{5}\cdot a^{\frac{3}{2}+1=\frac{5}{2}}+\frac{2}{3}\cdot a^{\frac{1}{2}+1=\frac{3}{2}}\right]+C$$